Compression of Images using Wavelets

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Abstract

Wavelet transformation is one of the most popular time-frequency-transformations. It is powerful feature for the signals and frequency analysis of an image. Image compression is key technology for transmission and storage of digital images. In this paper the compression algorithms are applied on various biomedical images. The algorithms used in this paper are SPIHT and EZW for comparing the quality of images after that the quality of images is compared by taking PSNR and MSE of images. Analysis of the quality measures have been carried out to reach to a conclusion.

Keywords- SPIHT, EZW, STFT, MSE, PSNR, GIF

I. INTRODUCTION

Image compression is important for many applications that have transmission, large data. Image compression is divided into two categories, lossless compression[1] and lossy compression. In lossless compression, the image can be recreated by using image information, there is no loss of information and this type of compression is also known as entropy coding. While in lossy compression, there is loss of information when the image information is reconstructed. In general, the image is required to fully reconstruct without loss. However, the lossy image is used for obtaining a higher compressed ratio.

Peak Signal to Noise Ratio (PSNR) is a quality metric, because increasing PSNR values indicates increasing reconstructed image fidelity. For a compressed image, the PSNR can be found as the number of bits used in the compressed representation of the image divided by the number of pixels in the image and it is measured in bits/pixel. The image can be compressed by using quality metric or resolving several distortion inorder to match the original image in lossy compression.

$$PSNR = 10 \log \left[ \frac{M \times N \times (2^B - 1)^2}{\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} [g(m,n) - f(m,n)]^2} \right]$$

Here M and N denote the image width and height, respectively; f(m,n) and g(m,n) denote original and reconstructed image; and B denotes the dynamic range(in bit) of the original image.

SPIHT[2] [3] algorithm was introduced by Said and Pearlman[4] and is improved and extended version of Embedded Zerotree Wavelet (EZW) coding algorithm introduced by Shapiro[5][6]. An image compression algorithm of higher compression ratio based on SPIHT is proposed and simulated, with studying on the SPIHT algorithm and image compression algorithm.

Firstly, the image data is divided into high frequency wavelet coefficients and low frequency wavelet coefficients, through the discrete wavelet transform. Secondly, the wavelet coefficients, which are processed and decomposed to several blocks, pass SPIHT coding, according to the compression ratio of image to adjust the SPIHT output coding stream. Finally, the original image is reconstructed by the SPIHT decoding algorithm and inverse wavelet transform. Simulating results show that a higher Peak Signal to Noise Ratio (PSNR) in proposed algorithm than traditional SPIHT algorithm is obtained in the same compression ratio of the image. Both algorithms work with tree structure, called Spatial Orientation Tree (SOT), that defines the spatial relationships among wavelet coefficients in different decomposition subbands. In this way, an efficient prediction of significance of coefficients based on significance of their parent coefficients is enabled.

SPIHT is a low-complexity progressive image compressor. This enhanced implementation of a zero-tree algorithm efficiently encodes zerotrees with a relatively modest level of complexity and produces an embedded bitstream. A higher image compression ratio can not be obtained by using SPIHT algorithm only, it can be obtained by using a lossy algorithm based on SPIHT encoding algorithm is proposed in this paper. Firstly, the wavelet coefficients are divided into several blocks, then the importance of different blocks are by adopting SPIHT algorithm of different blocks are encoded by adopting SPIHT algorithm of different bit rate, in order to improve the compression ratio.
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II. FRAME OF SIMULATING SYSTEM

According to the importance of the wavelet coefficients blocks, the simulation system consists of three parts: a Discrete Wavelet decomposition of image data, a SPHIT encode module which performs the coding of wavelet coefficients in the block by bit rate and a bit allocate module which controls the SPHIT encode module that input the bit stream as shown in the fig.1.

III. SPIHT ALGORITHM

One of the important features of SPIHT was designed for optimal progressive transmission, as well as for compression. The another important features of SPIHT is the quality of the displayed image is the best at any point during the decoding of an image, that can be achieved for the number of bits input by the decoder up to that moment

The wavelet coefficients can be referred as $c_{i,j}$. In a progressive transmission method, the decoder starts by setting the reconstruction image to zero. It then inputs (encoded) transform coefficients, decodes them, and uses them to generate an improved reconstruction image. The main aim in progressive transmission is to transmit the most important image information first. SPHIT uses the mean squared error (MSE) distortion measure.

$$\text{MSE} = \frac{\sum_{i=0}^{N-1} \sum_{j=0}^{N-1} (x_{i,j} - \hat{x}_{i,j})^2}{\sum_{i=0}^{N-1} \sum_{j=0}^{N-1} (x_{i,j})^2}.$$  

Where, $N$ is total number of pixels. So the largest coefficients contain the information that reduces the MSE distortion.
A. **SPIHT Coding**

It is important to have the encoder and decoder test sets for significance in the same way, so the coding algorithm uses three lists called list of significant pixels (LSP), list of insignificant pixels (LIP), and lists of insignificant sets (LIS).

1) **Initialization:** Set n to \(\log_2 \max_{i,j} |c_{i,j}|\) and transmit n. Set the LSP to empty. Set the LIP to the coordinates of all the roots \((i,j)_H\). Set the LIS to the coordinates of all the roots \((i,j)_H\) that descendants.

2) **Sorting pass**
   - for each entry \((i,j)\) in the LIP do:
     - output \(S_n(i,j)\);
     - if \(S_n(i,j) = 1\), move \((i,j)\) to the LSP and the sign of \(c_{i,j}\);
   - for each entry \((i,j)\) in LIS do:
     - if the entry is of type A, then
       - output \(S_n(D(i,j))\);
     - if \(S_n(D(i,j)) = 1\), then
       - for each \((k,l)_O(i,j)\) do:
         - output \(S_n(k,l)\);
         - if \(S_n(k,l) = 1\), add \((k,l)\) to the LSP,
         - output the sign of \(C_{k,l}\);
     - if \(L(i,j)\) not equal to 0, move \((i,j)\) to the end of the LIS, as a type-B entry, and go to step else, remove entry \((i,j)\) from the LIS;
   - if the entry is of type B, then
     - Output \(S_n(L(i,j))\);
     - if \(S_n(L(i,j)) = 1\), then
       - append each \((k,l)_O(i,j)\) to the LIS as a type-An entry:
       - Remove \((i,j)\) from the LIS;

3) **Refinement pass:** for each entry \((i,j)\) in the LSP, except those included in the last sorting pass (the one with the same n),
   - output the nth most significant bit of \(|c_{i,j}|\);

4) **Loop:** decrement n by 1 go to step 2 if needed

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### IV. EZW Algorithm

The EZW (Embedded Zero trees of Wavelet transforms) algorithm is the first and powerful algorithms based on Wavelet based Image compression. EZW of wavelet transforms is a lossy image compression algorithm. The algorithms were created depending upon the fundamental concepts of EZW. The EZW algorithm was introduced in this paper of Shapiro. The core of the EZW compression is the exploitation of self-similarity across different scales of an image wavelet transform. In other words EZW approximates higher frequency coefficients of a wavelet transformed image. By considering the transformed coefficients as a tree(ors trees) with lowest frequency coefficients at the root node and with the children of each tree node being the spatially related coefficients in the next frequency subband, there is a high probability that one or more subtrees will consists entirely of coefficients which are zero or nearly zero, such subtrees are called zerotrees. Zerotrees are used to represent the significance map in an efficient way. The main steps are as follows:

1) **Initialization:** Set the threshold \(T\) to the smallest power of 2 that is greater than \(\max_{i,j} |c_{i,j}| / 2\), where \(c_{i,j}\) are the wavelet coefficients.

2) **Significance map coding:** Scan all the coefficients in a predefined way and output a symbol when \(|c_{i,j}| > 2\). When the decoder inputs this symbol, it sets

3) **Refinement:** Refine each significant coefficient by sending one more bit of its binary representation. When the decoder receives this, it increments the current coefficient value by \(\pm 0.25T\).

4) **Set** \(T = T/2\), and go to step 2 if more iterations are needed.

#### A. EZW Coding

A Wavelet coefficient \(c_{i,j}\) is considered insignificant with respect to the current threshold \(T\) if \(|c_{i,j}| = T\). The zerotree data structure can be constructed from the following experimental result: If a wavelet coefficient at a coarse scale (i.e., high in the image pyramid) is insignificant with respect to a given threshold \(T\), then all of the coefficients of the same orientation in the same spatial location at finer scales (i.e., located lower in the pyramid) are likely to be insignificant with respect to \(T\). In each iteration, all the coefficients are scanned in the order shown in fig. 3. This guarantees that when a node is visited, all its parents will already have been scanned.

Each coefficient visited in the scan is classified as a zerotree root (ZTR), an isolated zero (IZ), positive significant (POS) or negative significant (NEG). A zerotree root is a coefficient that is insignificant and all its descendants (in the same
spatial orientation tree) are also insignificant. Such a coefficient becomes the root of a zerotree. It is encoded with a special symbol (decoded by ZTR). When the decoder inputs a ZTR symbol, it assigns a zero value to the coefficients and to all its descendants in spatial orientation tree. Their values get improved in subsequent iterations. The fig 4 illustrates this classification.

Fig. 3: Scanning of Zerotree

![Scanning of Zerotree](image)

Fig. 4: Classifying a Coefficient

![Classifying a Coefficient](image)

Two lists are used by the encoder (and also by the decoder, which works in lockstep) in the scanning process. The dominant list contains the coordinates of the coefficients that have not been found to be significant. They are stored in the order scan, by pyramid levels, and within each level by subbands. The subordinate list contains the magnitudes of the coefficients that have been found to be significant. Each list is scanned once per iteration. Iteration consists of a dominant pass followed by a subordinate pass. In the dominant pass, coefficients from the dominant lists are tested for significance. If a coefficient is found significant, then i) its sign is determined, ii) it is classified as either POS or NEG, iii) its magnitude is appended to the subordinate list, and iv) it is set to zero in memory (in the array containing all the wavelet coefficients). The last step is done so that the coefficient does not prevent the occurrence of a zerotree in subsequent dominants passes at smaller thresholds. At the end of the subordinate pass, the encoder sorts the magnitudes in the subordinate list in decreasing order. The encoder stops the loop when a certain condition is met and the decoder stops decoding when the maximum acceptable distortion has been reached.
V. EXPERIMENTS

The different bio medical images are used for experiments. PSNR (Peak Signal to Noise Ratio) values and MSE (Mean Square Error) values are found by using the above experiment result. The result that got by using SPHIT technique are shown:

A. Original Images

![Original Images](image1.png) ![Original Images](image2.png)

1) Compressed Images with SPHIT Algorithm and Various Wavelets

1) Image: Mri Liver_out_of_phase.gif

![Compressed Images](image3.png) ![Compressed Images](image4.png) ![Compressed Images](image5.png)
2) Image: Anatomic_imaging_of_the_shoulder_axia_t1_se.gif

Fig. 10: Sphit-Anatomic_imaging_of_shoulder.gif-bior4.4

Fig. 11: Sphit-Anatomic_imaging_of_shoulder.gif-Sym3

Fig. 12: Sphit-Anatomic_imaging_of_shoulder.gif-haar

2) Compressed Images with EZW Algorithm and Various Wavelets

Fig. 13: EZW-mriLiver_out_of_phase.gif-bior4.4

Fig. 14: EZW-mriLiver_out_of_phase.gif-haar
VI. Performance Analysis

The above algorithms are compared and performance is evaluated. The PSNR and MSE values for the images are calculated by using the following formula. It is shown in the table 1 and table 2.

$$\text{PSNR} = 10 \log_{10} \left( \frac{255^2}{\text{MSE}} \right)$$
The SPHIT method is the advance method for image compression. It requires keen attention because it provides highest image quality, progressive image transmission, fully embedded coded file. It is effective in broad range of reconstruction qualities because of its embedded coding process, simple quantization, exact bit rate coding and Error protection.

Table 1: PSNR and MSE Values for SPHIT

<table>
<thead>
<tr>
<th>Image</th>
<th>Wavelet</th>
<th>PSNR</th>
<th>MSE</th>
<th>AD</th>
<th>NAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>anatomic_imaging_of_the_shoulder_axial_tl_se.gif</td>
<td>Bior4.4</td>
<td>45.96</td>
<td>1.65</td>
<td>0.00</td>
<td>0.02</td>
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<tr>
<td>anatomic_imaging_of_the_shoulder_axial_tl_se.gif</td>
<td>Sym3</td>
<td>45.16</td>
<td>1.98</td>
<td>0.00</td>
<td>0.02</td>
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<tr>
<td>anatomic_imaging_of_the_shoulder_axial_tl_se.gif</td>
<td>haar</td>
<td>42.32</td>
<td>3.81</td>
<td>-0.00</td>
<td>0.03</td>
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<tr>
<td>mri_orbita_tl.gif</td>
<td>Bior4.4</td>
<td>37.06</td>
<td>12.80</td>
<td>0.02</td>
<td>0.04</td>
</tr>
<tr>
<td>mri_orbita_tl.gif</td>
<td>Sym3</td>
<td>36.37</td>
<td>14.99</td>
<td>0.02</td>
<td>0.04</td>
</tr>
<tr>
<td>mri_orbita_tl.gif</td>
<td>haar</td>
<td>34.72</td>
<td>21.94</td>
<td>0.02</td>
<td>0.05</td>
</tr>
<tr>
<td>mri_liver_out_of_phase.gif</td>
<td>Bior4.4</td>
<td>24.53</td>
<td>229.12</td>
<td>0.01</td>
<td>0.44</td>
</tr>
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<td>mri_liver_out_of_phase.gif</td>
<td>Sym3</td>
<td>45.02</td>
<td>2.05</td>
<td>0.01</td>
<td>0.03</td>
</tr>
<tr>
<td>mri_liver_out_of_phase.gif</td>
<td>haar</td>
<td>43.60</td>
<td>2.84</td>
<td>0.01</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Best and worst: Bior4.4 and haar

Table 2: PSNR and MSE Values for EZW

<table>
<thead>
<tr>
<th>Image</th>
<th>Wavelet</th>
<th>PSNR</th>
<th>MSE</th>
<th>BPP</th>
<th>AD</th>
<th>NAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>anatomic_imaging_of_the_shoulder_axial_tl_se.gif</td>
<td>Bior4.4</td>
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<td>22.53</td>
<td>0.06</td>
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<td>22.07</td>
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<td>0.07</td>
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<td>anatomic_imaging_of_the_shoulder_axial_tl_se.gif</td>
<td>haar</td>
<td>32.50</td>
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<td>0.09</td>
<td>-0.02</td>
<td>0.08</td>
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<td>Bior4.4</td>
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<td>0.09</td>
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<td>29.16</td>
<td>78.83</td>
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<td>0.02</td>
<td>0.09</td>
</tr>
<tr>
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<td>27.47</td>
<td>116.33</td>
<td>0.24</td>
<td>0.02</td>
<td>0.10</td>
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<td>Bior4.4</td>
<td>30.64</td>
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<td>51.34</td>
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<td>69.71</td>
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<td>-0.01</td>
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</tr>
</tbody>
</table>
VII. CONCLUSION AND FUTURE SCOPE

In this paper we have applied various wavelets of wavelet family on SPIHT as well as EZW and observed different results. By comparing we found that SPIHT shows better PSNR and MSE performances on the various test images. Our results show that different wavelets performed differently for different medical images, but generally the difference between each other was not that much. As the results shown Bior4.4 is well suited to anatomic_imaging_of_the_shoulder_axia_t1_se.gif, mri_orbita_t1.gif and haar is worst performing, but in case of mri_liver_out_of_phase_.gif Sym3 is best performing and bior4.4 is worst performing.

Similarly, when EZW compression algorithms is applied with different wavelets, the different wavelets performed differently for different medical images. As the results shown, Sym3 is well suited for image anatomic_imaging_of_the_shoulder_axia_t1_se.gif and Bior4.4 is best performing for image mri_orbita_t1.gif and again Sym3 is best performing for image mri_liver_out_of_phase.gif as in the SPHIT algorithm, but haar is performing worst for all images.

REFERENCES